San Diego Math Circle - Euler
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Combinatorics Combo Meal with a side of PIE

Hao Ye

Polyhedra

Polyhedron	Faces	# Edges	# Vertices
Tetrahedron	4 triangles		
Hexahedron (Cube)	6 squares		
Octahedron	8 triangles		
Dodecahedron	12 pentagons		
Icosahedron	20 triangles		
Truncated Icosahedron	20 hexagons, 12 pentagons		

Venn Diagrams

Each of the 39 students in the eighth grade at Lincoln Middle School has one dog or one cat or both a dog and a cat. 20 students have a dog and 26 students have a cat. How many students have both a dog and a cat?
AMC-8 2008.11

PIE (Principle of Inclusion-Exclusion)

For two sets, A and B, the union $|A \cup B| = |A| + |B| - |A \cap B|$.
For three sets, A, B, and C, the union $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$.

– have a brother
– have a sister
– have a pet

Basic Combinatorics

Standard California license plates have 7 characters, with 1 number, then 3 letters, then 3 numbers. The letters, I, O, and Q cannot be used in the first or third letter positions. How many different license plates are possible?

Fancier Combinations

How many 3-digit positive integers have digits whose product equals 24?
AMC-8 2009.16

Even Fancier Combinations

Three red beads, two white beads, and one blue bead are placed in line in random order. What is the probability that no two neighboring beads are the same color?
AMC-10B 2008.22

Stars and Bars

Seven distinct pieces of candy are to be distributed among three bags. The red bag and the blue bag must each receive at least one piece of candy; the white bag may remain empty. How many arrangements are possible?
AMC-10B 2010.22

Probability

8 points are spaced around at intervals of one unit around a 2 x 2 square, as shown. 2 of the 8 points are chosen at random. What is the probability that the 2 points are 1 unit apart?
AMC-8 2008.19